Geoid model of Uganda from airborne gravity survey 2020

René Forsberg, Hergeir Teitsson
National Space Institute, Technical University of Denmark (DTU-Space)
rf@space.dtu.dk

Background

A new geoid model for Uganda has been computed from new gravity data measured during the 2020 DTU-Space/MLHUD airborne gravity survey of Uganda. The airborne survey was sponsored by the US National Geospatial Agency, as part of improvements of the next global gravity field model EGM2020. The new geoid model (Fig. 1) represents a major improvement of the national geodetic infrastructure, allowing better height determination with GPS in connection with engineering, urban development, flood control and food security.

The new geoid model – *UGEOID2020A* – is expected to have an accuracy of 3-5 cm across much of Uganda, and is tailored to match the existing height system of Uganda through precise GPS survey of 7 primary levelling survey markers, fitting these points with an error of 2 cm r.m.s. The new geoid is based on the airborne survey data, the latest satellite data from the ESA GOCE mission through the global reference gravity field model XGM2019, terrain data from the Tandem-X satellite mission, as well as available gravity field data in the surrounding countries.

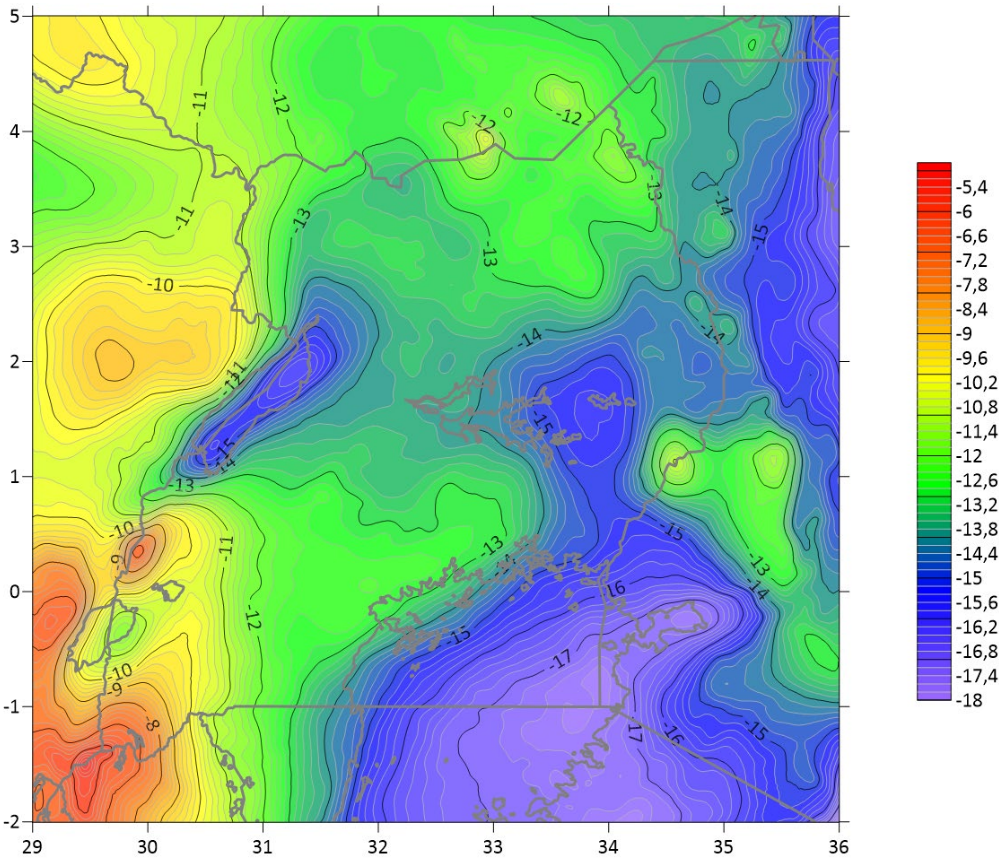

Fig. 1. UGEOID2020A – new geoid of Uganda from airborne survey. Contour interval 20 cm.

This note describes the methods, data and how to use the new geoid model in practice. The geoid is available in various grid formats, covering the region 2°S - 5°N, 29°- 36°E. The description of the airborne survey itself can be found in reference [1].

A significant amount of older surface gravity measurements in Uganda have in addition been used for an auxillary 2nd geoid model, after quality check and outlier rejection of the older (1950-60's) surface gravity data. This 2nd geoid model – *UGEOID2020B* – is potentially subject to likely data biases and errors in the older gravity data, giving rise to additional errors in the geoid. The "B" geoid model should therefore only be used in regions with reasonable coverage of the older data, e.g. in the Kampala region. An effort should be made to re-occupy some of the older gravity data points with new land gravimeter measurements and GPS; in this case older data could be "rescued" through re-computation of Bouguer anomalies, even if the old points cannot be relocated accurately; such data can make a future geoid model even more accurate. The "B" geoid model would in principle be equivalent to the UGQ2014 quasigeoid described in ref [2], except for differences in QC, editing and methodology, and the addition of the new airborne survey data.

1. Principle of geoid determination

A geoid model is a surface (N) which describes the theoretical height of the ocean and the corresponding zero-level surface on land. The geoid is required to obtain the height above sea level from GPS by

$$H = h^{GPS} - N \tag{1}$$

where h^{GPS} is the GPS ellipsoidal height, and H the levelled (orthometric) height. This equation is the classic equation for height determination with GPS. However, it only holds in a global system of reference, so to be consistent with local heights, the geoid need to be fitted to local heights, based on local sea level connections. This is in practice done by observing apparent local geoid heights, at points with both precise GPS and levelling heights, by

$$N^{GPS} = h^{GPS} - H \tag{2}$$

and then fitting an empirical surface to the differences

$$\varepsilon = N_{grav} - N^{GPS} \tag{3}$$

and modelling this difference across a larger area, giving a *GPS geoid*, i.e. a geoid which is consistent with the local height system. It should be noted that a GPS geoid is not a geoid in the classical definition (geoid is defined as an equipotential surface in the earth's gravity field), but rather a surface which is a composite of the gravimetric geoid, GPS and levelling, with all three elements contributing unavoidable errors. In the case of Uganda, the "ε" correction was adopted as a constant term, as sufficient GPS-levelling were not available across Uganda.

2. Gravimetric geoid computation of Uganda

The Uganda gravimetric geoid is computed by the *GRAVSOFT* system, a set of Fortran routines developed through many years of research and project work at DTU-Space and the Niels Bohr Institute, University of Copenhagen [3]. It forms the base of major recent geoid computation projects, such as the joint Nordic "NKG" geoid models, undertaken as joint geoid model computations of the Nordic and Baltic countries [4] under the auspices of the Nordic Commission for Geodesy (NKG), as well as the OSGM02 geoid model of the UK and Ireland [5], and several national geoid models done from airborne surveys in recent years (Malaysia, Mongolia, Indonesia, Tanzania, Mozambique), as well as recent IAG computation experiments (such as the international Colorado geoid inter-comparison experiment currently being published).

The Uganda geoid has been computed by a "remove-restore" technique, where a spherical harmonic earth geopotential model (EGM) is used as a base, and the geoid is computed from the global contribution N_{EGM}, a local gravity derived component N_2, and a terrain part N_3.

$$N_{grav} = N_{EGM} + N_2 + N_3 \tag{4}$$

The spherical harmonic expression as a function of latitude, longitude and height is of form

$$N(\phi, \lambda, r) = \frac{GM}{R\gamma} \sum_{n=2}^{N} \left(\frac{R}{r}\right)^n \sum_{m=0}^{n} \left(C_{nm} \cos m\lambda + S_{nm} \sin m\lambda\right) P_{nm}(\sin \phi) \tag{5}$$

where GM, R and γ are earth parameters. For the UGEOID2020 models, the newest reference model XGM2019 has been used, incorporating GOCE, GRACE and other satellite data, as well as global surface gravimetry from EGM2008, see http://icgem.gfz-potsdam.de/home. The XGM2019 is complete to degree and order 720, and may be seen as a precursor to the new NGA EGM2020 model for the longer wavelengths; reference system and satellite data planned to be the same.

In the XGM2019 model satellite data from GOCE and GRACE determine the error spectrum of the geoid up to spherical harmonic degree 180-200 or so. We use the model for the Uganda geoid only to degree 360, to limit the effect of the XGM2019 local gravimetry data errors (XGM2019 incorporates old terrestrial 5' average gravity data, which might have large errors). The role of the reference model is further diminished by the use of Stokes' kernel modifications, which determines the split between the influence of the reference model versus the new airborne gravity data, discussed in more details in the next section.

To fully represent the three-dimensional nature of the gravity field, the computations of geoid and gravity values of the XGM2019 was done in dense grids at two levels (0 and 4 km), and the use of a 3-D "sandwich grid" interpolation. In this way the XGM2019 geoid grids are actually quasigeoids at different levels, and the resulting XGM2019 "geoid" actually a quasigeoid.

The terrain part of the geoid computation is based a new 8" x 8" Digital Elevation Model (DEM), constructed from Tandem-X satellite interferometry data, edited for outliers in the major lakes (Lake Victoria and Lake Albert), and fill-in of some data voids with SRTM data. The final DEM is shown in Fig, 1, after conversion from ellipsoidal to orthometric heights.

The use of the DEM is based on the RTM terrain reduction method, where topography is referred to a mean elevation level, and only residuals relative to this level is taken into account. The 8" DEM was averaged and filtered with a circular filter of radius 0.283° radial cut-off to match the use of XGM2019 to degree 360.

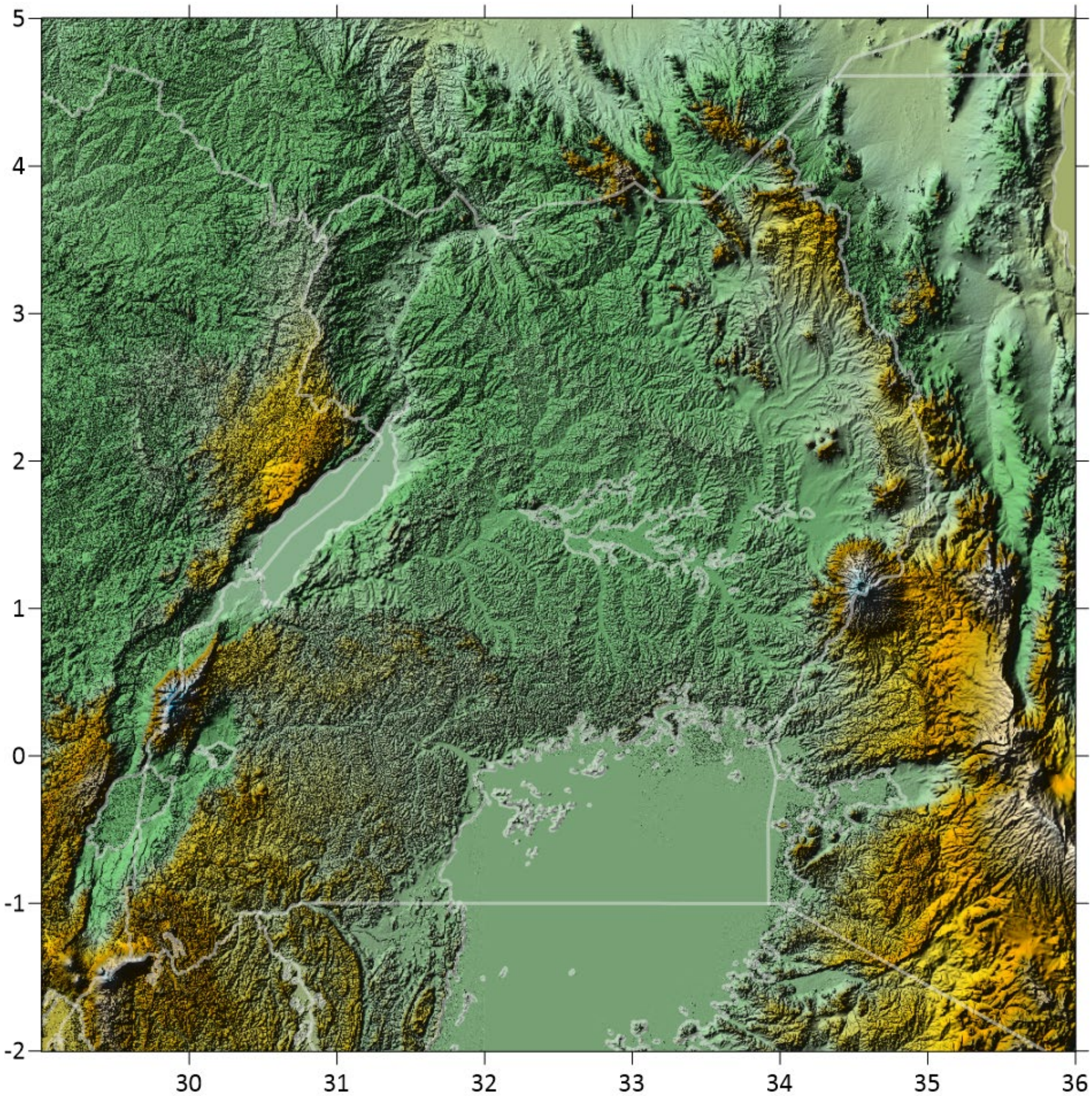

Figure 2. Tandem-X DEM of Uganda, used for the geoid determination. The rough terrain along the western border (Rwenzori mountains, up to 5109 m) and eastern border (Mt Elgon, 4321 m) is clearly seen, as is the major rift valley to the west with Lake Albert and Lake Edward, all sources of major geoid variability.

The method for the gravimetric geoid determination is *spherical FFT with optimized kernels.* This is a variant of the classical geoid integral ("Stokes integral"), in which there is a proper weighting of the long wavelengths from XGM2019 and the shorter wavelengths from the airborne and local gravity data. Mathematically it involves evaluating convolution expressions of form

$$N_2 = S_{ref}(\Delta\varphi, \Delta\lambda) * [\Delta g_2(\varphi, \lambda)\sin\varphi] = F^{-1}[F(S_{ref})F(\Delta g \sin\varphi)] \qquad (6)$$

Here S_{ref} is a modified "Stokes" kernel, $\Delta g_2 = \Delta g - \Delta g_{EGM}$ is the XGM2019-reduced free-air gravity anomalies, and F is the 2-dimensional Fourier transform operator. For details see references [4]-[6] and further references therein.

The Uganda geoid is computed on a grid of 1’ x 1’ resolution (corresponding to 1.8 km resolution). The computations have been based on least squares collocation and Fast Fourier Transformation methods. The FFT transformations at the 1’ resolution involve 840 x 840 grid points, including the necessary zero-padding for avoiding FFT errors along the borders of the grid. The data are gridded and downward continued by least squares collocation using the planar logarithmic model. A number of GRAVSOFT programs are involved in this process (*gpcol1, spfour, gcomb, geoip).*

Several geoid models were computed, either by simple set ups based on surface data only, or more elaborate setups incorporating full three-dimensional handling of airborne and surface gravity data. The selected final gravimetric geoid solution was computed by following steps:

- *Subtraction of XGM2019 spatial reference field (in a 3-D “sandwich mode”)*
- *RTM terrain reduction of surface and airborne gravimetry, after editing for outliers*
- *Downward continuation to the terrain level and gridding of all data by least-squares collocation using a 1° x 1° moving-block scheme with 0.6° overlap borders*
- *Spherical bandwise Fourier Transformation from gravity to geoid*
- *Restore of RTM and XGM2019 effects on the geoid to yield the quasigeoid*
- *Conversion of quasigeoid to classical geoid, using the linear Bouguer correction*

The above scheme is (advanced) standard methods of physical geodesy.

3. Gravity data used and QC for the geoid computation

The Uganda 2020 geoid models are based on the following data:

- *Airborne DTU Space gravity data from the Uganda 2020 survey. These data are free-air anomalies at the aircraft altitude, with atmospheric correction.*
- *Airborne DTU gravity data from Tanzania 2012, in the border regions to Uganda, with permission from the Surveys and Mapping Division, Dodoma.*
- *Land gravity data in Uganda, from the Bureau Gravimetrique (https://bgi.obs-mip.fr). These data are old, generally from the 1950-60’s era, and with many errors.*
- *PGM2017 5’ gravity grid data in neighbouring countries except Tanzania (PGM2017 an IAG restricted high-resolution spherical harmonic model, a precursor of EGM2020).*
- *Tandem-X DEM data.*
- *Long-wavelength data from XGM2019 (including latest GOCE and GRACE satellite data).*

Some plots of the used data are shown in the Figs. 3-7 below, and the statics of the various data reductions are shown in Table 1.

QC of the Uganda surface gravity data has been done by DTU Space in a QGIS environment using the fully terrain- and XGM2019 reduced data, for an example see Fig. 7. Many excessive outliers have been deleted in this process, and a major complete BGI source (#3480001, digitized Bouguer anomaly map of major parts of eastern Uganda) was found to be erroneous and altogether deleted. Some residual biases are still evident when compared to downward continued airborne data, especially in the Kampala region; it is recommended to check some of the older sources shown in Fig. 4 by new gravimeter/GPS measurements for future improvements in “joint” geoid models based both on surface and airborne data. The large bias of 5.5 mGal in the reduced data mean value in Table 1 also point to terrestrial data bias problems (mean ideally should be ~zero).

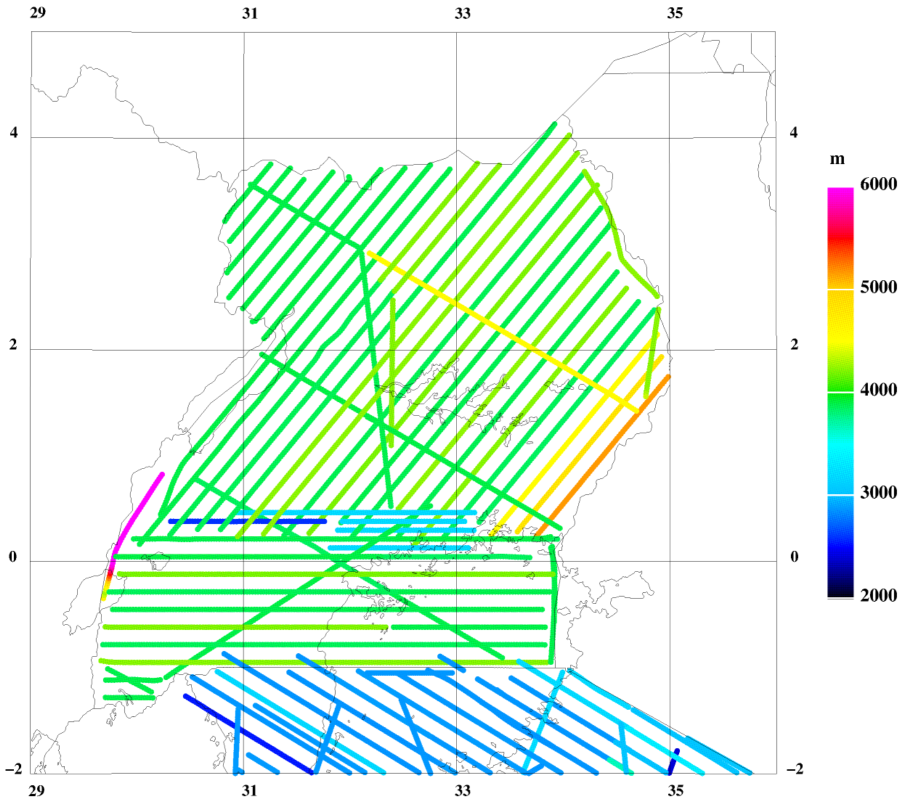

Figure 3. Airborne data used for the geoid. Colours show flight elevations. A flight level around 4000 m was needed in Uganda to clear terrain and clouds. Lower level flights around Kampala done to improve geoid.

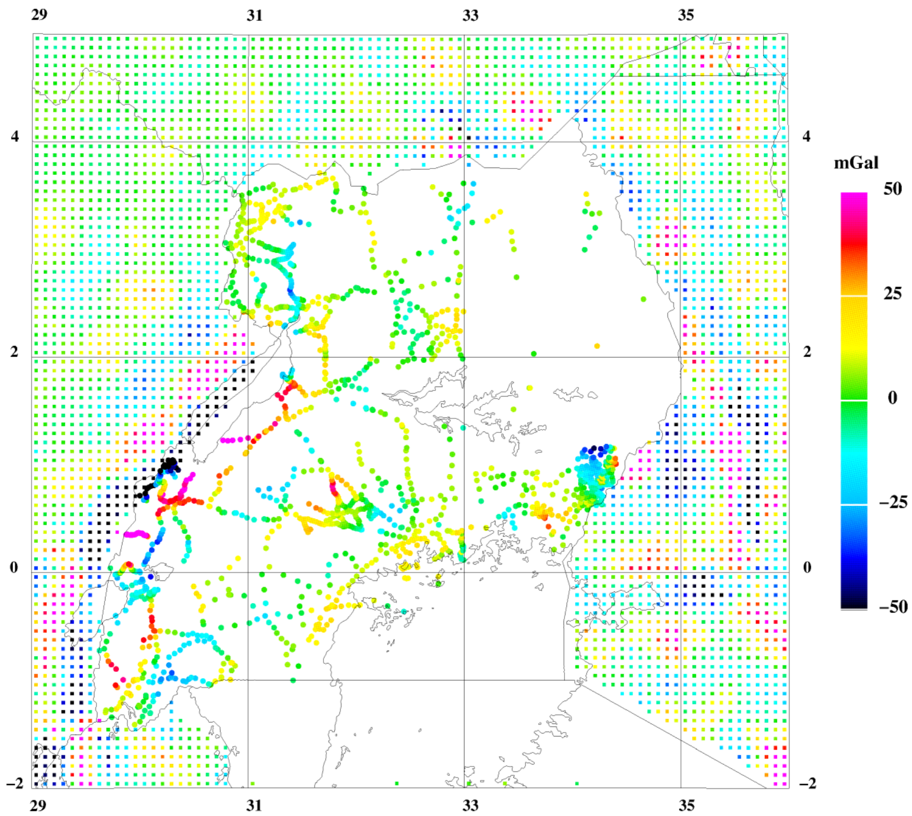

Figure 4. Edited BGI surface gravimetry in Uganda, and used PGM2017 gridded free-air anomalies in neighbouring countries. Colours shows the fully reduced free-air anomaly data. 47% of BGI data were QC-deleted.

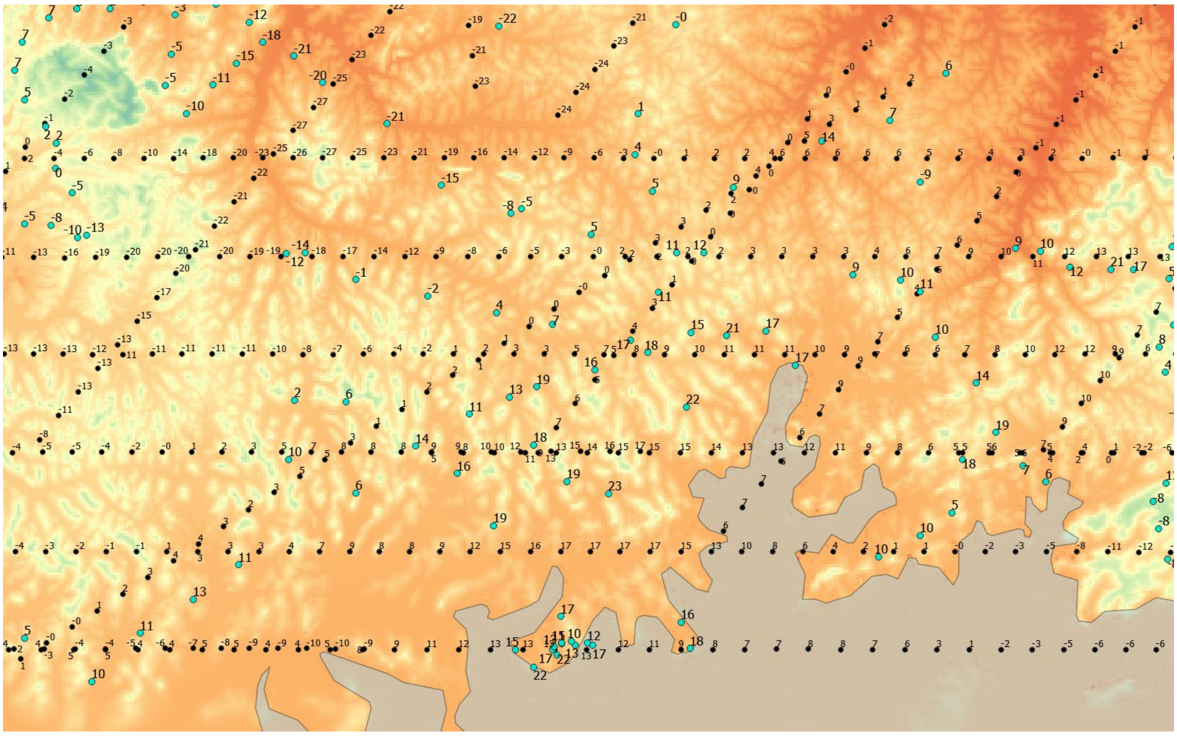

Figure 5. Reduced gravity anomalies from airborne survey (black dots) and BGI edited data (grey dots). There appears to be a systematic offset of a few mGal between the two data sources, and also a small end-of-line problem for one of the cross-lines in the airborne survey (trimmed in the final geoid).

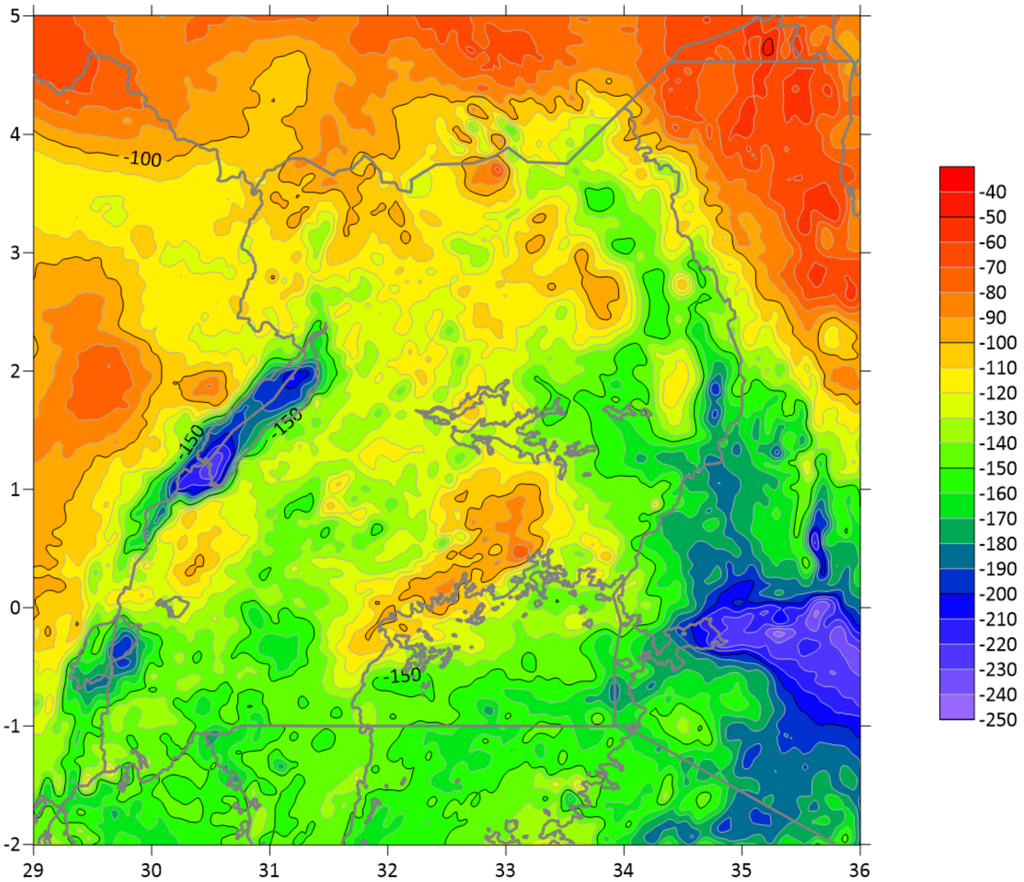

Fig. 6. Bouguer anomalies from the downward continued airborne and the surface data. Unit mGal, density 2.67 g/cm^3. The Bouguer anomalies are used in geology, but also for the geoid-quasigeoid correction.

Table 1. Statistics of remove steps in the Uganda gravimetric geoid computation

Unit: mGal	Mean	Std.dev.	Min	Max
Edited land gravity data (1611 points)	-0.2	39.3	-148.4	323.1
Land gravity minus XGM2019 and RTM	5.5	25.6	-76.1	218.2
Airborne gravity (21971 pts @ 10 sec interval)	-3.8	26.0	-117.3	229.3
Airborne minus XGM2019 and RTM	0.4	13.3	-47.2	145.5

4. Geoid processing results

The plots in the sequel shows the intermediate results of the final remove-restore geoid processing, computed with full 3-dimensional modelling, going via the quasigeoid to the classical, final UGEOID2020A geoid for Uganda, shown in Fig. 1.

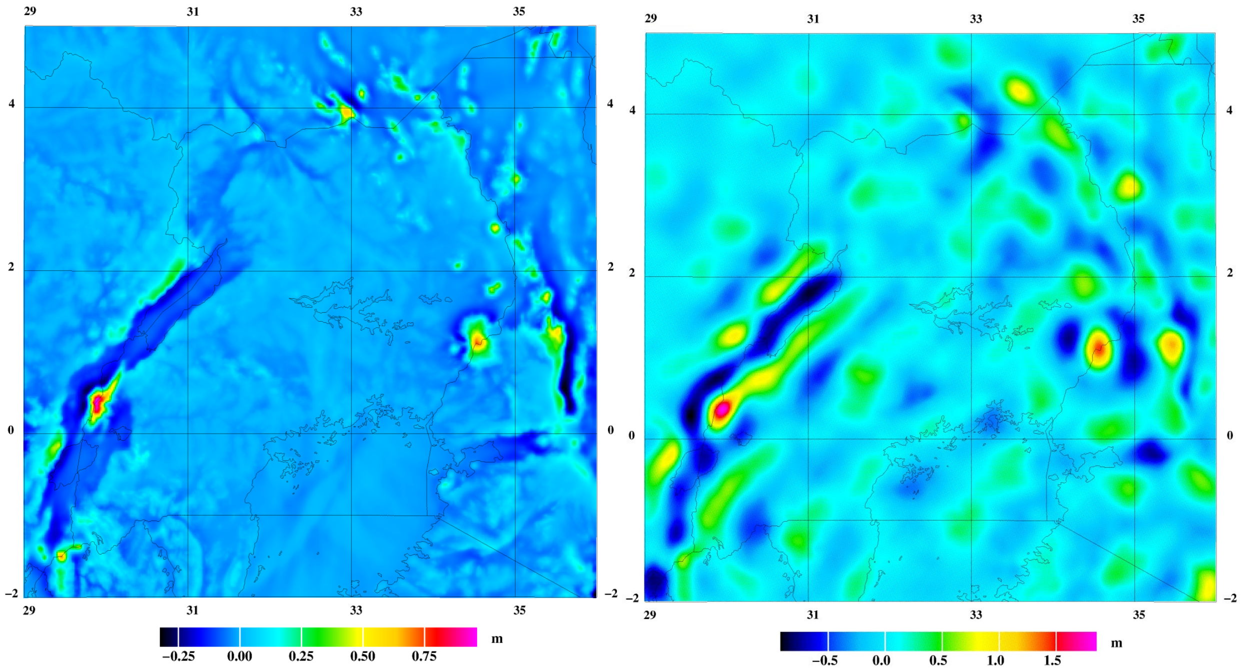

Fig. 7. The geoid “restore signals. Left: the RTM terrain effects (N_3), highlighting the mountainous regions. Right: The geoid component coming from the airborne gravity data (N_2), as outlined in formula (4).

The downward continuation step from gravity anomalies are dependent on the covariance parameter, and after some iterations of balancing the fit between surface and airborne data, a set of parameters $\sqrt{C_0}$ = 15 mGal, D = 5 km, T = 40 km was used for the planar logarithmic covariance function, for details see [6]. The parameters imply a correlation length of 15 km for reduced surface gravity. Since the covariance function is only used for the downward continuation and gridding of gravity, the process is quite insensitive to the selection of these parameters. Both the airborne and surface gravity were assigned an apriori noise of 2 mGal in this process. The subsequent modified Stokes kernel transition was set to degree 120 after some GPS-levelling comparisons, a quite low value, but in agreement with experience from other regions.

The magnitude of the last steps in the gravimetric geoid determination (the conversion of quasigeoid to geoid) is shown in Figure 8, and the difference between the two geoids “A” and “B” are shown in Fig. 9. It can be seen that the use of the terrestrial gravity has a quite large impact.

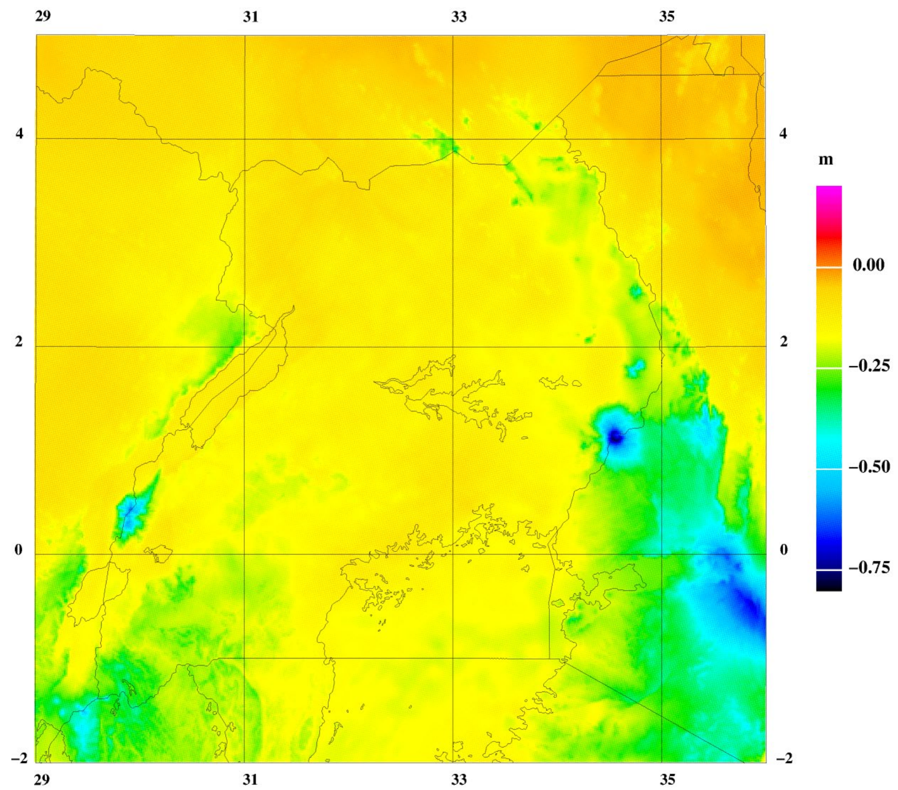

Fig. 8. The difference between geoid and quasigeoid, computed from the DEM and Bouguer anomalies. The correction is quite significant, also in Kampala region, but of course dominated by the mountains regions.

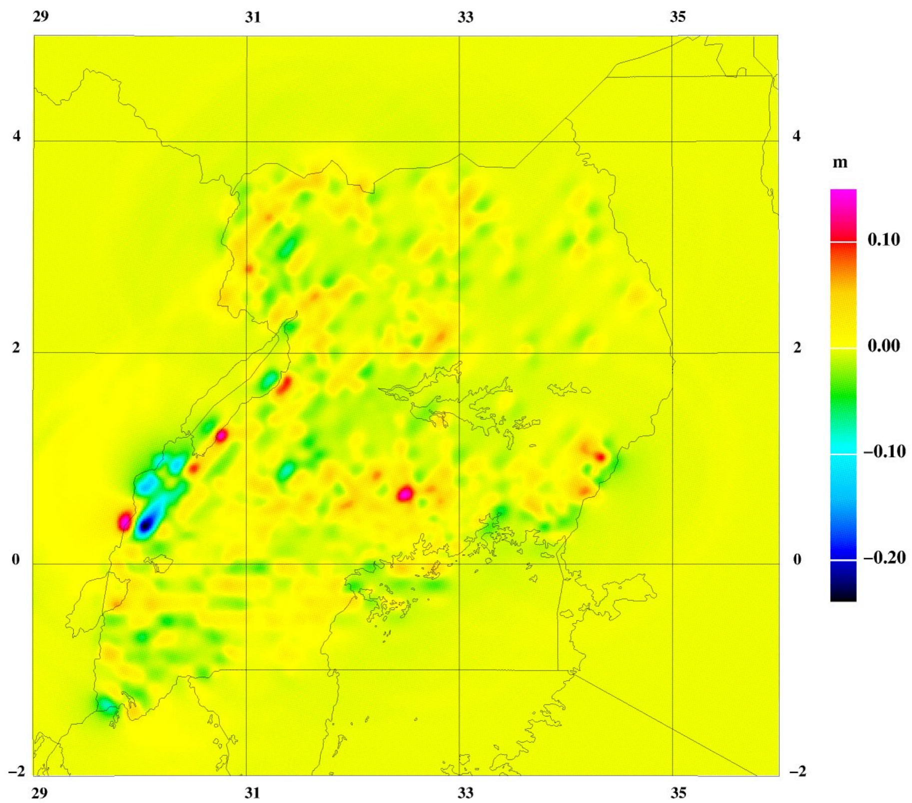

Fig. 9. The difference between UGEOID2020B (airborne and surface gravity data) and UGEOID2020A (airborne data only). The differences are especially large along the airborne lines, and some outliers.

5. Comparison to GPS-levelling data

Only two GPS-levelling data sets ($N^{GPS} = h^{GPS} - H_{levelling}$) were available for a validation of the Uganda geoid, and the determination of the geoid shift needed for consistency with the national New Khartoum height datum of Uganda. A set of 7 precise GPS positions of precise levelling points were given by Abeho et al. [ref 7], and a set of 12 GPS levelling points in Kampala were given by Kyamulesire et al. [ref 8]. The location of the 7-station data set is shown in Fig 10; this data set was computed with very long occupations (48 to 144 hours) and processed with GLOBK/GAMIT, with indicated GPS accuracies below 1 cm for the ellipsoidal heights.

The 12-point local data sets in Kampala, spanning mainly an approximately 15 km long profile, was similarly tied to a to a global GPS reference system by using the AUSPOS positioning service for one of the network points, and with the use of Bernese software should be quite accurate too. But a common point to the 7 station network showed a discrepance of approximately 32 cm in the derived N^{GPS} value, so the 12-point network was not used in the transformation definition. Comparison results are shown in Table 2.

Figure 10. The location of the 7 GPS/levelling points, used to define the offsets of the Uganda GPS geoid.

Table 2. Comparison of GPS levelling data sets to the Uganda geoids (N^{GPS}-$N_{gravimetric}$)

Unit: m	7-station net		12-station Kampala	
	Mean	Std.dev.	Mean	Std.dev
Gravimetric geoid, airborne only	.347	.020	.594	.098
Gravimetric geoid, airborne and surface Δg	.663	.037	.575	.095
UGEOID2020A (airborne only)	**0**	**.020**	**.247**	**.098**
UGEOID2020B (surface and airborne)	-.005	.037	.228	.095
Test geoid, surface data only	*.543*	*.128*	*.362*	*.077*
EIGEN-6C4 quasigeoid	*.244*	*.169*	*.427*	*.076*

It is quite amazing how well the 7-station network fit the "A" geoid, confirming a 5 cm error estimate of the geoid (1-σ) in most regions. It is also surprising to some degree that the Kampala network seems to be better with the surface data, than the airborne data alone. This could point to some residual noise in the airborne data or be a coincidence in a local area, or the effect of the relative high flight levels; more gravimetry data around Kampala could be useful to check the underlying gravity data around Kampala. More GPS/levelling data, with long occupations on 1st order levelling points, could also be very useful to qualify errors and validate the geoid(s).

Table 2 also shows the fit the EIGEN-6C4 geoid, as used in the recent GPS network modernization of Uganda. As there are many variants of the EIGEN-6C4 geoid (quasigeoid, geoid, reference system used), and it is not known which variant was used in the GPS modernization, the EIGEN-6C4 comparison is done by directly evaluating the spherical harmonic model (6) to degree 2160 at sea level, in the WGS84 system (Fig. 11), i.e. as a quasi-geoid at sea level. This model fit the refence point "Rhino Camp", one of the 7 defining GPS points, with an offset of 58.3 cm.

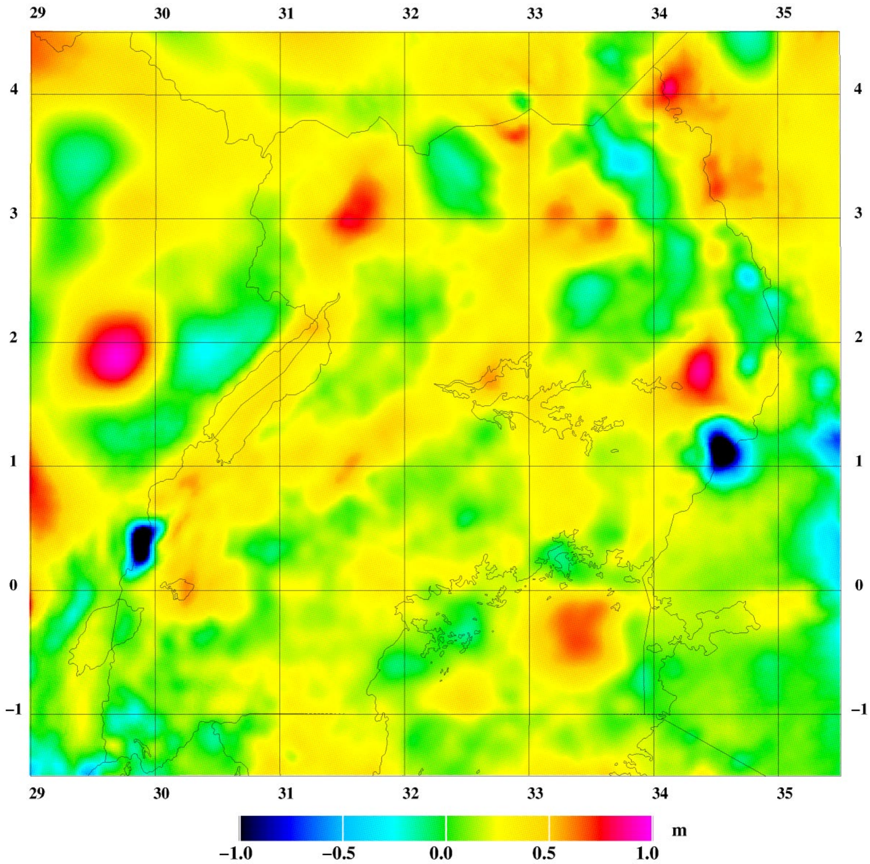

Fig. 11. Difference between EIGEN-6C4 quasigeoid and the UGEOID2020A.

The overall conclusion of the validation is that the Uganda geoid has an amazingly good fit across the large 7-station network, without any kind of fitting other than a bias fit. Therefore the UGEOID2020A is recommended for use as the new GPS/Khartoum height system for Uganda.

6. Interpolation tool for the Uganda geoid

The Uganda geoids are given in the native GRAVSOFT format, i.e. stored in E-W rows from N to S, initiated by a label defining the area, as shown below for UGEOID2020A (first lines in file):

```
-2.000000    5.000000   29.000000   36.000000  0.01666667  0.01666667

  -10.211     -10.202     -10.195     -10.186     -10.180     -10.171
  -10.164     -10.157     -10.150     -10.141     -10.135     -10.129
  -10.123     -10.120     -10.119     -10.118     -10.120     -10.122
  -10.121     -10.127     -10.133     -10.137     -10.147     -10.157
  -10.167     -10.182     -10.196     -10.209     -10.224     -10.238
   ....
```

The geoid is also made available in geotiff (.tif), Surfer (.grd), Leica (.leica) and Trimble (.ggf) GPS software formats. The Leica format is a simple lat/lon/geoid file.

A Python visual tool (*geoid_int.exe*) is available for interpolating individual points in either *decimal degree* or *[deg,min,sec]* formats, as shown below in Fig. 12. The program may also interpolate ascii files in *[id, lat, lon, height]* format point files, and convert between from ellipsoidal to orthometric heights or reverse (this option only supports positions in decimal degrees).

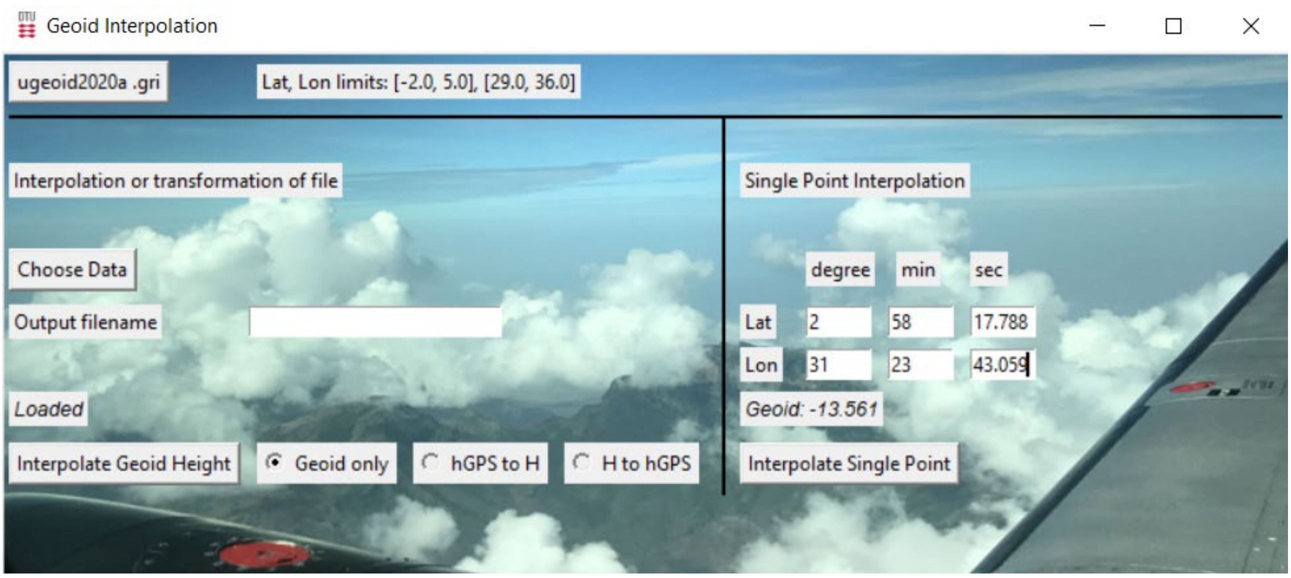

Fig 11

The 7 defining points N^{GPS} and $N_{UGEOID2020A}$ are listed below for checks of interpolation:

	LAT	LON	h_GPS	H	N_GPS	UGeoid2020A
JINJA	0 25 08.69758	33 12 00.39103	1162.498	1176.064	-13.566	-13.550
71Y97	0 20 17.73450	32 33 53.31548	1255.314	1267.942	-12.628	-12.641
KASESE	0 10 46.06511	30 04 37.67715	980.896	990.380	-9.484	-9.487
KIBOGA	0 54 52.41815	31 46 09.67010	1174.464	1187.649	-13.185	-13.158
MBARARA	-0 36 33.68073	30 39 12.62674	1448.208	1459.021	-10.813	-10.835
MUBENDE	0 33 43.59150	31 23 24.85620	1311.438	1324.002	-12.564	-12.585
RHINO C	2 58 17.78812	31 23 43.05879	617.809	631.387	-13.578	-13.561

References

[1] Rene Forsberg, Hergeir Teitsson, Adolfientje Kasenda: Airborne Gravity Survey of Uganda 2020 - Acquisition and processing report. Technical report, National Space Institute, Technical University of Denmark (DTU-Space), May 2020.
[2] Ronald Ssengendo: A height datum for Uganda based on a gravimetric quasigeoid model and GNSS/levelling. Ph.d. thesis, Royal Institute of Technology, Stockholm, Sweden, 2015.
[3] Forsberg, R and C C Tscherning: Overview manual for the GRAVSOFT Geodetic Gravity Field Modelling Programs, 3rd edition. Technical report, DTU-Space, 2014.
[4] Forsberg, R., D. Solheim and J. Kaminskis: Geoid of the Nordic and Baltic area from gravimetry and satellite altimetry. Proc. Int. Symposium on Gravity, Geoid and Marine Geodesy, Tokyo, Sept. 1996, pp. 540-548, Springer Verlag IAG Series, 1996.
[5] Forsberg, R., G. Strykowski, J.C. Illife, M. Ziebart, P.A.Cross, C.C. Tscherning , P. Cruddace, K. Stewart, C. Bray and O. Finch: OSGM02: A new geoid model of the British Isles. Proceedings of the 3rd meeting of the International Gravity and Geoid Comission, GG2002, Aug. 26 - 30, 2002, Thessaloniki, I. Tziavos (ed.), Editions Ziti, pp. 132-137, 2003.
[6] Forsberg, R. and A. Olesen: Forsberg, Airborne gravity field determination. In: G. Xu (ed): Sciences of Geodesy – I, Advances and Future Directions, pp. 83-104, Springer Verlag, 2010, ISBN 978-3-642-11741-1.
[7] Dianah Rose Abeho, Roger Hipkin and Bedada Besha Tulu: Evaluation of EGM2008 by means of GPS Levelling Uganda. South African Journal of Geomatics, Vol. 3, No. 3, November 2014.
[8] Kyamulesire, B., Oluyori, P. D. and Eteje, S. O.: Comparative Analysis of Three Plane Geometric Geoid Surfaces for Orthometric Height Modelling in Kampala, Uganda. FUDMA Journal of Sciences, Vol. 4 No. 3, September, 2020, pp 48 – 51, DOI: https://doi.org/10.33003/fjs-2020-0403-255.